Fractionally spaced blind equalization: CMA versus Second Order based methods.

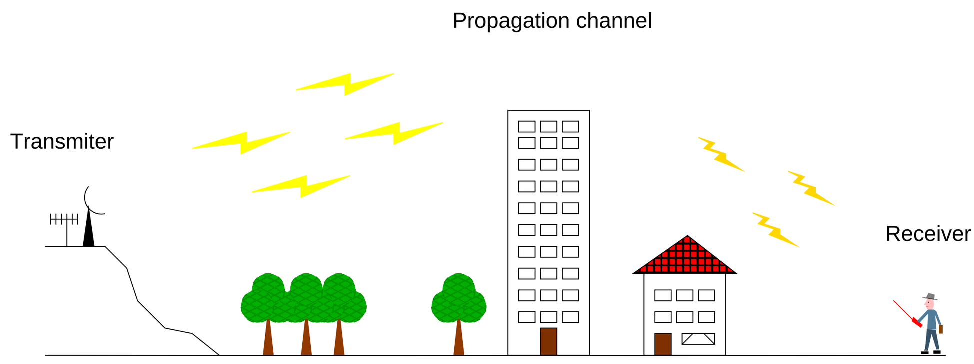
L. Mazet, Ph. Ciblat and Ph. Loubaton
Laboratoire Système de Communication
Université de Marne-la-Vallée
email: (mazet, ciblat, loubaton)@univ-mlv.fr

5, boulevard Descartes
Champs sur Marne
77454 Marne-la-Vallée Cedex 2, France
Tel.: (33) 1 60 95 72 90 Fax: (33) 1 60 95 72 14

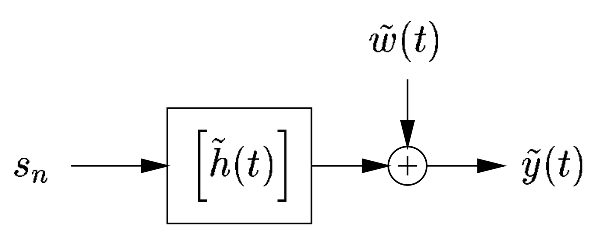
The work of the first and second authors are respectively supported by CELAR and DGA/CNRS fellowship

Introduction

We consider a wireless communication problem.



The analogical equivalent system is :



with

- $(s_n)_{n\in\mathbb{Z}}$ a zero mean unit variance i.i.d. symbol sequence transmitted at baud rate $\frac{1}{T_s}$.
- $\tilde{h}(t)$ results from the shaping filter and a multipath propagation channel.
- $\tilde{w}(t)$ a white noise.
- $\tilde{y}(t)$ the analogical received signal.

Purpose of blind equalization :
Retrieve $(s_n)_{n\in\mathbb{Z}}$ without any knowledge of the channel from the $\tilde{y}(t)$ estimated statistics.
Our work :
Compare second order based methods with a fourth order based method (the CMA).

We choose to oversample $\tilde{y}(t)$ in respect of the second order based methods.
The discret equivalent system is :

$$\mathbf{y}(n) = [\mathbf{h}(z)]\, s_n + \mathbf{w}(n)$$

with

- $\mathbf{y}(n) = \left[\tilde{y}(2n\frac{T_s}{2}), \tilde{y}((2n+1)\frac{T_s}{2})\right]^{\mathrm{T}}$ (it is a 2-variate discrete time signal).
- $\mathbf{w}(n) = \left[\tilde{w}(2n\frac{T_s}{2}), \tilde{w}((2n+1)\frac{T_s}{2})\right]^{\mathrm{T}}$.
- $\mathbf{h}_k = \left[\tilde{h}(2k\frac{T_s}{2}), \tilde{h}((2k+1)\frac{T_s}{2})\right]^{\mathrm{T}}$.
- $\mathbf{h}(z)=\sum_{k=0}^{M}\mathbf{h}_k z^{-k}$.

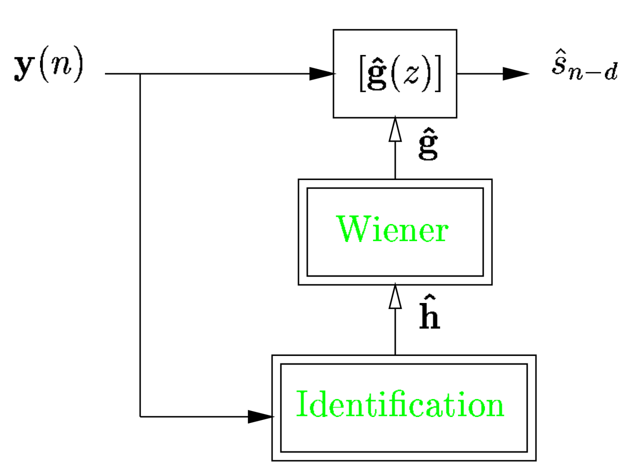
We denote $h(z)$ the scalar filter given by $h(z) = \sum_{k=0}^{2M+1}\tilde{h}(2k\frac{T_s}{2})z^{-k}$.
$\Longrightarrow$ $h(z)$ is band limited.

Compared methods

Second order based methods

1. Subspace method (SSM) introduced by [2]
$\Longrightarrow$ Poor performances if $h(z)$ is band limited. [1]
2. Optimally weighted covariance matching (CM).
$\Longrightarrow$ the best second order statistics based method to estimate $h(z)$.

After estimate of $h(z)$, we need to equalize our received signal.
$\Longrightarrow$ We choose a Wiener equalizer.

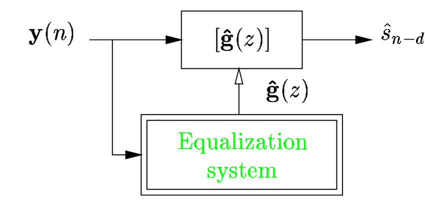


$\longrightarrow$ We have theorical results.

Fourth Order based method

1. Constant Modulus Algorithm (CMA)
$\Longrightarrow$ The most standard higher order statistics based method.

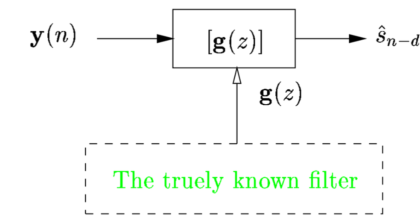
Provide $\hat{\mathbf{g}}(z)$ an equalizer estimate.



$\longrightarrow$ We have practical results.

Comparison with a non blind equalization scheme

Wiener equalizer computes with the full knowledge of $\mathbf{h}(z)$.



The covariance matching estimate.

Let $\mathbf{Y}_N(n)$ be $\begin{bmatrix}\mathbf{y}(n)\\ \vdots \\ \mathbf{y}(n-N)\end{bmatrix}$. Define $\mathbf{R}_N(\mathbf{h})$ the covariance matrix:

$$\mathbf{R}_N(\mathbf{h}) = \mathcal{T}_N(\mathbf{h})\mathcal{T}_N(\mathbf{h})^* + \sigma^2 I$$

where

- $\mathbf{h} = (\mathbf{h}_0^{\mathrm{T}}, \ldots, \mathbf{h}_M^{\mathrm{T}})^{\mathrm{T}}$
- σ^2 is the known noise variance.
- $\mathcal{T}_N(\mathbf{h})$ is the generalized Sylvester.

Denote $\tilde{\mathbf{R}}_N$ the empirical estimate of $\mathbf{R}_N(\mathbf{h})$.

$$\tilde{\mathbf{R}}_N = \frac{1}{T}\sum_{n=0}^{T-1}\mathbf{Y}_N(n)\mathbf{Y}_N(n)^*$$

Principle :
Look for a filter $\mathbf{f}(z)$ for which the matrix $\mathbf{R}_N(\mathbf{f})$ is as close as possible from the estimate $\tilde{\mathbf{R}}_N$.

$$\hat{\mathbf{h}}_W = \arg\min_{\mathbf{f}} \left\| \mathbf{W}^{\frac{1}{2}} \begin{bmatrix} \mathrm{vec}(\tilde{\mathbf{R}}_N) - \mathrm{vec}(\mathbf{R}_N(\mathbf{f})) \\ \mathrm{vec}(\overline{\tilde{\mathbf{R}}}_N) - \mathrm{vec}(\overline{\mathbf{R}}_N(\mathbf{f})) \end{bmatrix} \right\|^2$$

where $\mathbf{W}$ is a positive hermitian weighted matrix. It is well known that,

$$T \begin{bmatrix} \mathrm{vec}(\tilde{\mathbf{R}}_N) - \mathrm{vec}(\mathbf{R}_N(\mathbf{h})) \\ \mathrm{vec}(\overline{\tilde{\mathbf{R}}}_N) - \mathrm{vec}(\overline{\mathbf{R}}_N(\mathbf{h})) \end{bmatrix} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \mathcal{C}_{\mathbf{R}_N})$$

As $\mathbf{h}$ is a complex vector, we obtain that,

$$T \begin{bmatrix} \hat{\mathbf{h}}_W - \mathbf{h} \\ \overline{\hat{\mathbf{h}}}_W - \overline{\mathbf{h}} \end{bmatrix} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_{\mathbf{W}})$$

with the asymptotic covariance matrix $\Sigma_{\mathbf{W}}$ given by:

$$\Sigma_{\mathbf{W}} = [\mathbf{G}^\star\mathbf{W}\mathbf{G}]^{\#}\,\mathbf{G}^\star\mathbf{W}\mathcal{C}_{\mathbf{R}_N}\mathbf{W}\mathbf{G}\,[\mathbf{G}^\star\mathbf{W}\mathbf{G}]^{\#}$$

where the matrix $\mathbf{G}$ equals

$$\mathbf{G} = \begin{bmatrix} \left.\frac{\partial \mathrm{vec}(\mathbf{R}_N(\mathbf{f}))}{\partial \mathrm{vec}(\mathbf{f})}\right|_{\mathbf{f}=\mathbf{h}} & \left.\frac{\partial \mathrm{vec}(\mathbf{R}_N(\mathbf{f}))}{\partial \mathrm{vec}(\overline{\mathbf{f}})}\right|_{\mathbf{f}=\mathbf{h}} \\ \left.\frac{\partial \mathrm{vec}(\overline{\mathbf{R}}_N(\mathbf{f}))}{\partial \mathrm{vec}(\mathbf{f})}\right|_{\mathbf{f}=\mathbf{h}} & \left.\frac{\partial \mathrm{vec}(\overline{\mathbf{R}}_N(\mathbf{f}))}{\partial \mathrm{vec}(\overline{\mathbf{f}})}\right|_{\mathbf{f}=\mathbf{h}} \end{bmatrix}$$

The optimal weight $\mathbf{W}$ is $\mathbf{W}_{opt} = \mathcal{C}_{\mathbf{R}_N}^{\#}$
(# stands for Moore-Penrose pseudo-inverse)

Consequences

- The optimal weighted matrix depends on $\mathbf{h}$.
- The cost function is not convex and admits a lot of local minima.

$\Longrightarrow$ Not easy for practical computation.

Analysis of the reconstruction error provided by a Wiener equalizer based on the covariance matching estimate.

For a known channel $\mathbf{h}$, the Wiener equalizer is the 1×2 FIR filter $\mathbf{g}(z) = \sum_{k=0}^{N}\mathbf{g}_k z^{-k}$ minimizing Γ defined by:

$$\Gamma = \mathbf{E}\left[\|v_{n-d} - [\mathbf{g}(z)]\mathbf{y}(n)\|^2\right] \quad \Longrightarrow \quad \mathbf{g} = \mathbf{h}^* P \mathbf{R}_N^{-1}$$

with $\mathbf{g} = (\mathbf{g}_0, \ldots, \mathbf{g}_N)$ and P is a certain selection/permutation matrix.

In practice, $\mathbf{h}$ and $\mathbf{R}_N$ unknown.
$\Longrightarrow$ we only get an estimate of the Wiener equalizer denoted $\hat{\mathbf{g}}(z)$.

$$\hat{\mathbf{g}} = \hat{\mathbf{h}}^* P \hat{\mathbf{R}}_N^{-1}$$

where $\hat{\mathbf{R}}_N = \mathcal{T}_N(\hat{\mathbf{h}})\mathcal{T}_N(\hat{\mathbf{h}})^* + \sigma^2 I$.
We evaluate

$$\Gamma = \mathbf{E}\left[\|v_{n-d} - [\hat{\mathbf{g}}(z)]\,\mathbf{y}(n)\|^2\right]$$

$\Longrightarrow$ Γ is the reconstruction error of the symbol sequence.

Assumptions :

- the Wiener filter is independent from the data.
- $\hat{\mathbf{h}} \to \hat{\mathbf{g}}$ is differentiable.

Result :

$$T(\hat{\mathbf{g}} - \mathbf{g}) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \mathcal{C}_{\mathbf{g}})$$

with the asymptotic covariance matrix $\mathcal{C}_{\mathbf{g}}$ given by:

$$\mathcal{C}_{\mathbf{g}} = \mathcal{D}_{\mathbf{g}}\Sigma_{\mathbf{W}_{opt}}\mathcal{D}_{\mathbf{g}}^*$$

where

$$\mathcal{D}_{\mathbf{g}} = \begin{bmatrix} \frac{\partial \hat{\mathbf{g}}}{\partial \hat{\mathbf{h}}} \\ \frac{\partial \hat{\mathbf{g}}}{\partial \overline{\hat{\mathbf{h}}}} \end{bmatrix}_{\hat{\mathbf{h}}=\mathbf{h}}$$

We obtain

$$\Gamma = \underbrace{\mathbf{E}\left[\|v_{n-d} - [\mathbf{g}(z)]\,\mathbf{y}(n)\|^2\right]}_{\text{Inherent Wiener filter reconstruction error}} + \underbrace{\mathbf{E}\left[\|[\Delta\hat{\mathbf{g}}(z)]\,\mathbf{y}(n)\|^2\right]}_{\text{Error due to } \mathbf{h} \text{ estimate}}$$

which implies

$$\Gamma = 1 - \mathrm{vec}(\mathbf{h})^\star P \mathbf{R}_N^{-1} P^\star \mathrm{vec}(\mathbf{h}) + \mathrm{Trace}\{\mathcal{C}_{\mathbf{g}}\mathbf{R}_N\}$$

Remark :
$\Longrightarrow$ Similar calculation for subspace method (only $\Sigma_{\mathbf{W}_{opt}}$ changes).

Conclusion

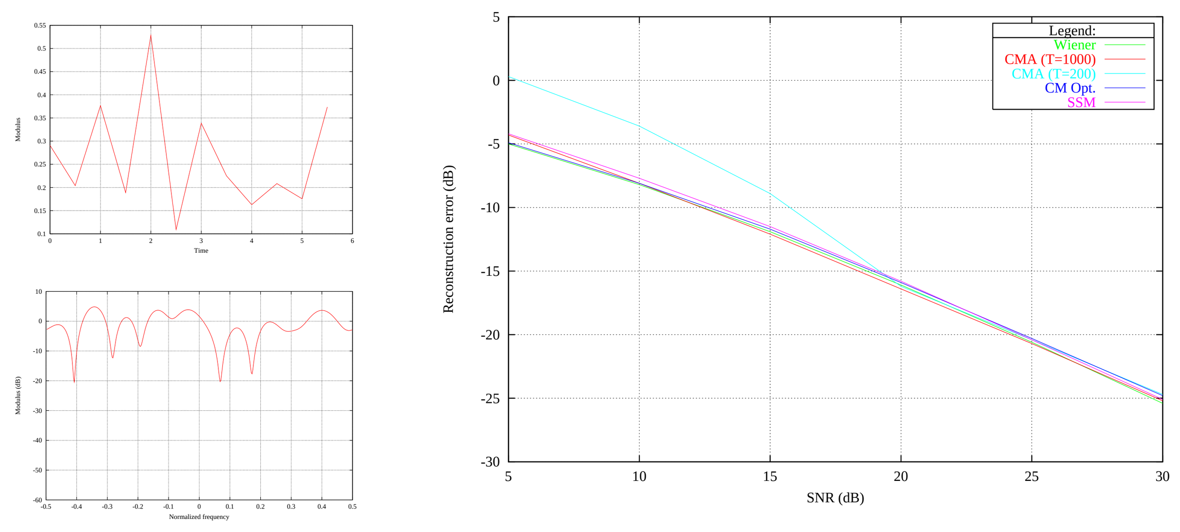
- We can obtain theorical results for the reconstruction error of the symbol sequence for the subspace and covariance matching methods.
- For CMA, only pratical results.

Simulations results

Reconstruction error of the symbol sequence versus SNR.

A random channel

- Random channel filter with 7 components.
- PSK-4 modulation.



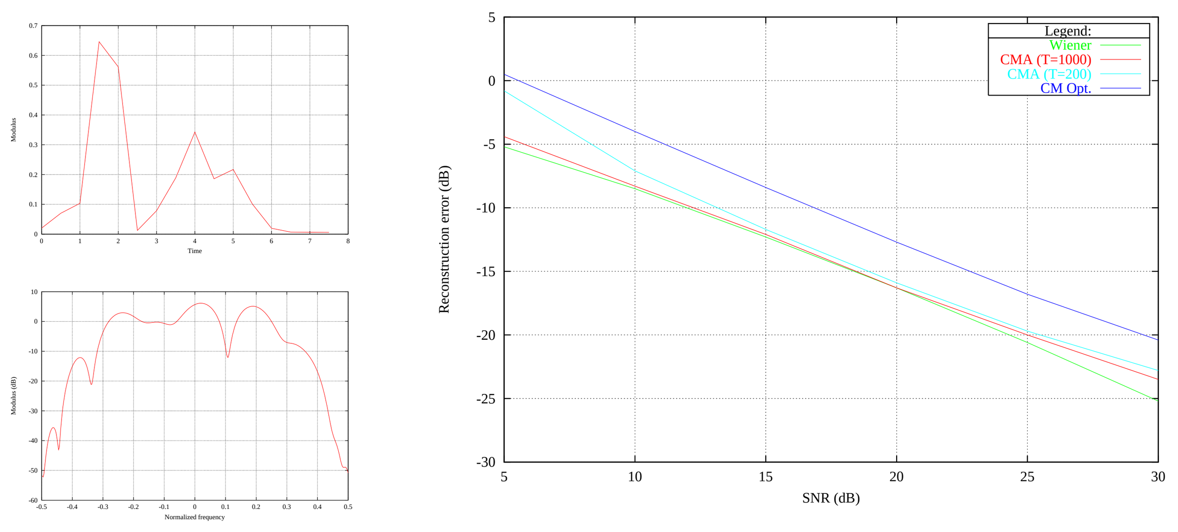
$\Longrightarrow$ All the schemes have quite the same performances.

Two realistic channels

Our shaping filter is a square root raised cosine filter with roll-off 0.7. $\Longrightarrow$ $h(z)$ is band limited.

Constant modulus modulation

- Propagation channel given by the following figures.
- PSK-4 modulation



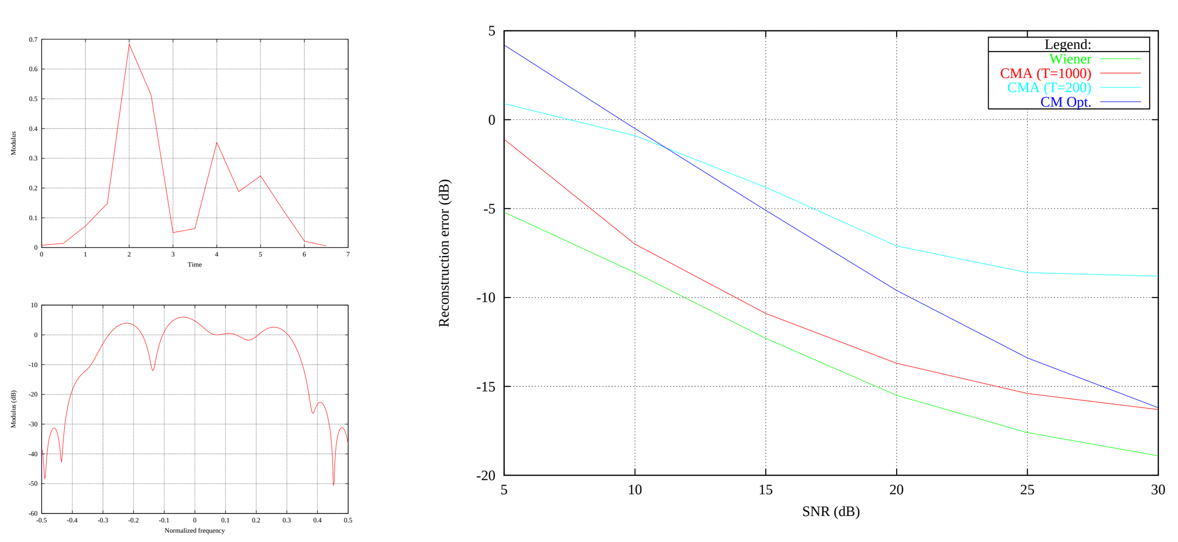
SNR (dB)	5.0	10.0	15.0	20.0	25.0	30.0
SSM / Wiener (dB)	64.7	56.8	47.8	38.2	28.3	18.4

We remark that,

- The subspace channel estimate gives extremely poor performance.
- The CMA outperforms the optimal second order scheme.
- The CMA performance is very close from the lower bound corresponding to the exact Wiener filter.

Non-constant modulus modulation

- Propagation channel given by the following figures.
- QAM-16 modulation



SNR (dB)	5.0	10.0	15.0	20.0	25.0	30.0
SSM / Wiener (dB)	49.5	41.5	32.5	23.0	13.3	3.7

We remark that,

- For 200 sized blocs, the CMA falls down due to non constant modulus modulation.
- For 1000 sized blocs, the CMA still outperforms optimally weighted covariance matching scheme.

Conclusion

$\Longrightarrow$ The covariance matching method considerably outperforms the subspace method.
$\Longrightarrow$ Standard pratical CMA equalizer produces better reconstruction errors than the theorical optimally weighted covariance matching.

References

[1] Ph. Ciblat, Ph. Loubaton, "Second order blind equalization: the band limited case", in *Proc. ICASSP 98*, vol. 6, pp. 3401-3404, Seattle, 1998.

[2] E. Moulines, P. Duhamel, J.F. Cardoso, S. Mayrargue, "Subspace method for the blind equalization of multichannel FIR filters" *IEEE Trans. Signal Processing*, vol. 43, pp. 516-526, February 1995.